

Mathematics**Time: 3 Hours****Total Marks: 100**

- Answer all Questions.
- Draw Diagrams wherever necessary.

Essay**(2x20=40)**

- Find $\lim_{x \rightarrow 2} \frac{x^3 - 2x^2}{x^2 - 5x + 6}$.
 - Find the derivative of $\frac{\cos x}{1 + \sin x}$.
 - Find $\int \frac{\tan^{-1} x}{1 + x^2} dx$.
- Find the constant λ so that the vectors $\vec{a} = 2\vec{i} - \vec{j} + \vec{k}$, $\vec{b} = \vec{i} + 2\vec{j} - 3\vec{k}$, $\vec{c} = 3\vec{i} + \lambda\vec{j} + 5\vec{k}$ are coplanar.
 - If $\vec{r} = \vec{a} \sin \omega t + \vec{b} \cos \omega t$ where $\vec{a}$, $\vec{b}$, ω are constants, show that $\frac{d^2 \vec{r}}{dt^2} = -\omega^2 \vec{r}$ and $\vec{r} \times \frac{d\vec{r}}{dt} = -\omega(\vec{a} \times \vec{b})$.
 - Find $\text{curl} \vec{F}$ where $\vec{F} = 2xy^3\vec{i} + x^2z^3\vec{j} + 3x^2yz^2\vec{k}$.

Short notes:**(8x5=40)**

- If $A = \begin{bmatrix} 2 & 7 \\ 1 & 2 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 6 \\ 9 & 2 & 5 \end{bmatrix}$ find AB.
- The sum of three consecutive numbers in an arithmetic progression is 18 and their product is 192. Find the numbers.
- Prove that $\frac{\sin \theta}{1 - \cos \theta} + \frac{\tan \theta}{1 + \cos \theta} = \sec \theta \operatorname{cosec} \theta + \cot \theta$.
- Prove that $\sin 75^\circ - \sin 15^\circ = \cos 105^\circ + \cos 15^\circ$.
- If $u = x^2 + y^2 + z^2$ prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 2u$.
- If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, show that $\operatorname{grad} \vec{r} = \frac{\vec{r}}{r}$.
- Find the modulus and amplitude of the complex number $\frac{1+2i}{1-3i}$.
- If the mean of a binomial distribution is 3 and variance is $\frac{3}{2}$, find the probability of obtaining at least 4 successes.

Answer briefly:

(10x2=20)

11. Using Simpson's $1/3$ rd rule solve $\int_0^6 \frac{1}{1+x} dx$.
12. Solve $\frac{dy}{dx} = e^{2x+3y}$.
13. In a class of 10 students, 4 are boys and the rest are girls. Find the probability that one selected student will be a girl?
14. The mean value of 25 items was 78.4. Later on it was discovered that one value was misread as 69 instead of the correct value 96. Calculate the correct average.
15. Find the cube roots of unity.
16. Find the divergence of the vector $\vec{v} = xyz\vec{i} + 3x^2y\vec{j} + (xz^2 - y^2z)\vec{k}$ at (2,-1,1).
17. Find $\frac{dy}{dx}$ if $y + \sin y = \cos x$.
18. Find $\int (\sin x + \cos x) dx$.
19. Find the principal value of $\sin^{-1}(\frac{1}{\sqrt{2}})$.
20. Find the value of $9P4$.
